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The determination of the lateral compliances and Poisson's ratios for highly oriented polyethylene sheets

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The extensional and lateral compliances have been determined for a series of oriented linear polyethylene sheets. For the measurements the sheet samples were mounted in a dead-loading creep apparatus fitted with a Hall effect lateral extensometer. The results agree well with those obtained previously on similar materials using other techniques. The values of the elastic constants are considered in the light of theoretical estimates based on force constant calculations.

1. Introduction

In recent years a number of publications have described the determination of the elastic constants for oriented polymers, either in the form of fibres or films. Comprehensive results have been presented for several polymers including polyethylene terephthalate [1], nylon [2] and low density polyethylene with the special "parallel lamellae" texture [3, 4]. In these cases it has been possible to use the results from a combination of techniques to obtain some understanding of the mechanical anisotropy in terms of the particular structure of the polymer.

The preparation of ultra high modulus oriented polymers [5] provides a new range of materials of interest. The present paper is one of several in which particular aspects of the mechanical anisotropy are discussed. A previous publication from these laboratories [6] presented preliminary data on the lateral compliance (and hence Poisson's ratio) of a specimen of ultra drawn high density polyethylene with a draw ratio, λ, in the range 20 to 24. The technique used was based on a Michelson interferometer which was used to measure the change in thickness of a stretched tape. In order to compensate for concomitant changes in refractive index (the dominant effect) the measurements had to be repeated with the specimen immersed in a liquid of known refractive index. Because of this problem and because of the small changes in thickness occurring at low strains in specimens approximately 100 μm in thickness the data were (a) not precise and (b) obtained at the comparatively high strain level of 0.5% where it is recognized that the mechanical response is non-linear.

The present paper is concerned with further measurements of the lateral compliances and Poisson's ratios of a series of polyethylene tapes using a specially constructed lateral extensometer which utilizes the Hall effect to detect the very small displacements. Apart from the intrinsic interest in the results from the viewpoint of the mechanical anisotropy in polymers, the Poisson's ratios are useful for design calculations in possible applications of these high modulus materials, especially in the reinforcement of brittle matrices.

2. Definition of elastic constants

The behaviour of an anisotropic solid is described by the generalized Hooke's law relating strains, e_i, to stress, σ_j, that is

$$e_i = S_{ij}\sigma_j \tag{1}$$

or

$$\sigma_i = C_{ij}e_j, \tag{2}$$

where S_{ij}, C_{ij} are the compliance and stiffness constants, respectively. Because we are applying constant stress and observe the corresponding strains it is most convenient to work in terms of the compliance constants. The high modulus polyethylene (LPE) sheets show orthohombic symmetry. We choose Cartesian axes such that the 3-

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TABLE I Details of polymer sheets studied

Polymer grade	Draw ratio, λ	Width (mm)	Thickness (mm)	Draw temperature (°C)
Hizex 7000F	6	70	0.16	90
Rigidex 50	8	16	0.22	120
Rigidex 50	12	16	0.23	92
Rigidex 50	15	15	0.16	90
Rigidex 50	24	15	0.08 to 0.10	112

axis is along the draw direction and the 1-axis is in the plane of the sheet perpendicular to the draw direction. There are nine independent elastic constants of which we are concerned with the extensional compliances S_{33} and S_{11} along the 3 and 1 directions respectively, and the lateral compliances S_{23} and S_{12} which correspond to changes in the thickness of the sheet for stresses applied along the 3 and 1 directions respectively. The corresponding Poisson's ratios are $\nu_{23} = -S_{23}/S_{33}$ and $\nu_{12} = -S_{12}/S_{11}$.

3. Experimental procedure

3.1. Materials

Oriented sheets of LPE were prepared by tensile drawing of extruded sheet at temperatures between 90 and 120° C. Further details of the materials are given in Table I.

The wide-angle X-ray diffraction patterns of the sheets showed that for samples with a draw ratio of 6, those with the incident X-ray beam in the plane of the sheet and normal to the plane of the sheet were indistinguishable, indicating that these possess transverse isotropy. All the higher draw ratio materials showed differences, indicating that there is an approach to the *b–c* sheet texture, with the *b*-axis preferentially oriented in the plane of the sheet normal to the draw direction, with increasing draw ratio.

For the determination of extensional and lateral compliances, samples in the form of 10 cm long strips were prepared with microtomed edges. The width of the strips varied between 2 and 3.5 mm. The cross-sectional areas were determined by measuring the width and thickness with a travelling microscope and a ball-end micrometer respectively.

3.2. Determination of extensional and lateral compliances

The extensional and lateral compliances were measured simultaneously by mounting the sample in a dead-loading creep machine fitted with a Hall effect lateral extensometer.

As full details of the lateral extensometer have already been published [4], it is only necessary to present a brief summary here. The thin polymer strip is extended between two Alnico permanent magnets mounted in a brass tube. The like poles of the magnets are adjacent, so that the magnetic field between the magnets has a null point. and the field gradient is twice that of a single magnet. The polymer strip is held in contact with one magnet by a stainless steel plate which contains the Hall effect device, and is covered by a thin stainless steel plate. The Hall plate is positioned so that the magnetic field sensing element is on the axis of the magnets. Continuous contact between the plate containing the Hall effect device and the specimen is ensured by exerting a small lateral compressive force by means of two pairs of phosphor bronze leaf springs.

4. Results

There are several features of the present work which require particular emphasis before we can begin to discuss the results.

Firstly, it was necessary to carry out a conditioning procedure before reliable results could be obtained. This procedure consisted of subjecting the sheets to successive loading and unloading cycles at a stress which gave rise to ~ 0.8% extensional strain at 10 secs, allowing 100 secs for recovery.

Secondly, determination of the isochronal 10 sec stress–strain curves for extensional strain, showed that these materials display very non-linear behaviour. The results are summarised in Fig. 1 where the 10 sec creep modulus is shown as a function of extensional strain for all samples. The non-linearity is most marked for the samples of highest stiffness.

Fig. 2 shows the extensional and lateral strains as a function of the applied stress for a sample of draw ratio 8. In order to compare with most publications from these laboratories we have collected data from all samples at the strain level of 0.1% and these are shown in Table II and Fig. 3.

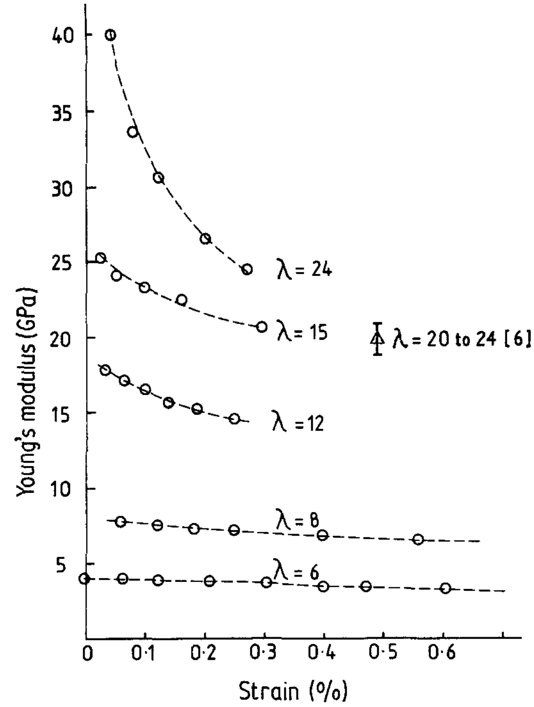

Figure 1 10 sec creep modulus as a function of extensional strain for samples of different draw ratio, λ.

Each of the compliance values was obtained from the mean of measurements taken on at least three samples at the strain 0.1%. Measurements indicated that the Poisson's ratio, $\nu_{23} = \epsilon_2(t)/\epsilon_3(t)$, was independent of strain within the scatter of the data (as seen in Fig. 2). The tabulated values are therefore also averaged over the entire strain range studied for each draw ratio (as indicated in Fig. 1).

In the sample of lowest draw ratio ($\lambda = 6$) it was possible also to apply the stress in the 1 direction so that S_{11}, S_{12} and hence ν_{12} could be determined. These results are shown in Table III.

5. Discussion

5.1. Comparison with previous results

The present results can be compared with those obtained in previous studies of oriented HDPE, which are summarised in Table IV. The results for the draw ratio of 6 sheets compare reasonably well with those for oriented HDPE monofilaments where S_{23} was determined by direct observation of the lateral contraction using an optical microscope [7]. The results for the high draw ratio sheets are very comparable to those for similar but thinner sheets where the lateral compliance was determined using a Michelson interferometer [6]. To compare the current data with those from the Michelson technique (corrected as in Table IV) the following points should be noted:

(a) The samples tested were produced by different processes and are therefore not strictly comparable.

(b) The data from the Michelson technique were obtained at a strain level of 0.5% (cf $\epsilon_3 =$ 0.1% in the present publication). Converting the extensional compliance into a Young's modulus enables the data to be compared directly with the present data as shown by the point marked Δ on Fig. 1. The Poisson's ratios are not considered to be strain dependent and so there is a small discrep-

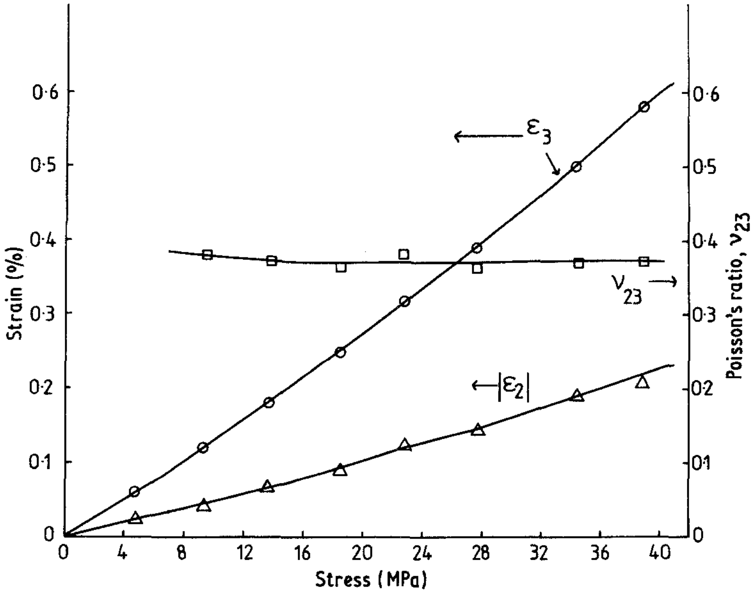

Figure 2 Extensional and lateral strain and Poisson's ratio, ν_{23}, as a function of stress for a sample of draw ratio 8.

TABLE II 10 sec isochronal compliances at 0.1% extensional strain

λ	S_{33} ($\times 10^{-10}$ Pa^{-1})	Young's modulus (GPa)	ν_{23}	S_{23} ($\times 10^{-10}$ Pa^{-1})
6	0.56 ± 0.33	3.9 ± 0.5	0.35 ± 0.05	1.01 ± 0.06
8	1.23 ± 0.07	8.15 ± 0.6	0.33 ± 0.05	0.355 ± 0.02
12	0.60 ± 0.015	16.75 ± 0.40	0.46 ± 0.03	0.285 ± 0.01
15	0.429 ± 0.01	23.85 ± 0.65	0.37 ± 0.12	0.176 ± 0.02
24	0.316 ± 0.012	32.6 ± 1.2	0.59 ± 0.08	0.193 ± 0.02

ancy between the results from the two techniques.

The Hall effect technique has some advantages over the latter technique in that it is not necessary to have highly transparent and homogeneous specimens with surfaces which are parallel and smooth to about one quarter of a wavelength of viable light over lengths of several millimetres. The interferometer technique also requires measurements of the fringe shift in two fluids, with the bulk of the fringe shift being due to the refractive index change as discussed above. The Hall effect technique does not have these disadvantages, and it is also easy to set up an analogue output to measure the time dependence and non-linearity. Its disadvantages are friction of the Hall plate against the specimen, the temperature sensitivity of the Hall effect and problems due to specimen twisting which are not as easily monitored as in an optical technique. It is clear from the present results that the two techniques are of comparable sensitivity but that the Hall effect method is of somewhat wider applicability as it does not require transparent samples.

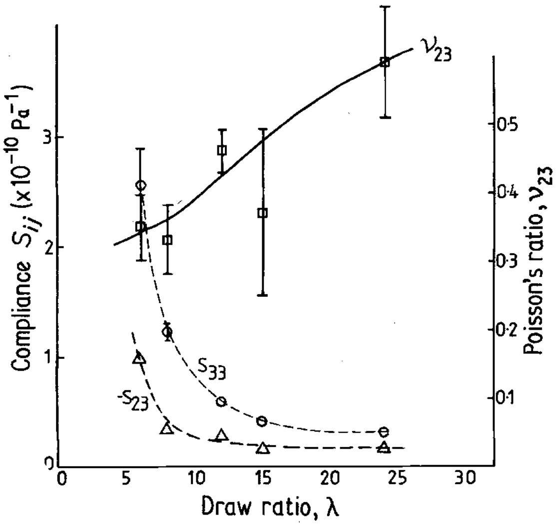

Figure 3 Extensional and lateral compliance and Poisson's ratio, ν_{23}, as a function of draw ratio (results for extensional strain of 0.1%.

5.2. Interpretation of mechanical anisotropy

In Table IV we can also compare the measured values of the elastic constants with theoretical estimates by Odajima and Maeda [8] and Tashiro *et al.* [9] based on force constant calculations. It can be seen that although the LPE sheets are of comparatively high stiffness even the minimum extensional compliance S_{33} is still about ten times the estimated theoretical value. The observed overall mechanical anisotropy is, however, consistent with that anticipated on theoretical grounds. In physical terms, the large anisotropy arises because S_{33} involves some element of bond stretching and bond bending whereas S_{11} is associated with dispersion forces. It also follows from such considerations that when the polymer is stressed in a direction perpendicular to the draw direction, the major contraction would be expected to take place in the 2 direction, perpendicular to the draw direction, rather than in the 3 direction which is parallel to the draw direction. Thus we would expect the magnitude of S_{12} to be greater than that of S_{23}, as is observed. This argument is similar to that invoked previously to explain the anisotropy of Poisson's ratios in polyethylene terephthalate [10], and explains why a value of $\nu_{12} = 0.62$, $\nu_{23} \sim 0.4$ is physically reasonable.

The theoretical estimates of ν_{12} and ν_{23} show a wide range, and indicate that these quantities are very sensitive to assumptions of the force-field calculations. In particular, Tashiro *et al.* [9] predict a very low value of -0.006 for S_{23} which does not seem physically reasonable. The values for ν_{12} in the range 0.13 to 0.33 also do not appear very reasonable in physical terms as explained above and are at variance with the experimental data. However, not too much emphasis should be placed on these discrepancies between the theoretical values for Poisson's ratios and those obtained experimentally. In view of the complicated nature of the theoretical calculations it is a considerable achievement to obtain estimates for the compliances which are reasonably

TABLE III 10 sec isochronal compliances at 0.4% extensional strain for polyethylene sheet, draw ratio 6:1

S_{23} (× 10^{-10} Pa^{-1})	E_0(= $1/S_{33}$)(GPa)	ν_{23}	S_{23} (× 10^{-10} Pa^{-1})	S_{11}	E_{90} (GPa)	ν_{12}	S_{12}
2.94 ± 0.17	3.24 ± 0.2	0.34 ± 0.05	1.0 ± 0.2	4.67 ± 0.5	21 ± 0.2	0.62 ± 0.07	2.95 ± 0.2

TABLE IV Collected data and comparison with theoretical estimates of crystal compliances

Material	S_{33} (× 10^{-10} Pa^{-1})	$-S_{23}$ (× 10^{-10} Pa^{-1})	ν_{23}	S_{11} (× 10^{-10} Pa^{-1})	$-S_{12}$ (× 10^{-10} Pa^{-1})	ν_{12}	Comments
HDPE, λ = 24	0.316 ± 0.012	0.193 ± 0.01	0.59 ± 0.08				$\epsilon = 0.1\%$, from this investigation
HDPE, λ = 20 to 24	0.50 ± 0.01	0.20 ± 0.05	0.42 ± 0.05				$\epsilon = 0.5\%$, from [6] (corrected figures)
HDPE, λ = 6	2.94 ± 0.17	1.0 ± 0.2	0.34 ± 0.05	4.76 ± 0.5	2.95 ± 0.2	0.62 ± 0.07	$\epsilon = 0.4\%$, from this investigation
HDPE, λ ~ 6 to 10	2.3 ± 0.3	0.77 ± 0.3	0.33 ± 0.12	15 ± 1	16 ± 2	1.1 ± 0.14	From Hadley, Pinnock and Ward [7] (fibres)
Theoretical crystal compliances	0.040 0.040 0.032	0.025 0.023 0.006	0.63 0.58 0.19	2.14 1.74 1.45	0.28 0.40 0.48	0.13 0.23 0.33	From Odajima and Maeda [8] From Tashiro, Kobayashi and Tadokoro [9]

close to the true values so that expressing the calculated results in terms of Poisson's ratio which is a ratio of compliances is an unreasonably critical test.

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